THE TERNARY DESCRIPTION LANGUAGE AS A FORMALISM FOR THE PARAMETRIC GENERAL SYSTEMS THEORY : PART II

AVENIR I. UYEMOV

Odessa University, Odessa 65000, Ukraine

This part is a continuation of the first part of my article that was published in *International Journal of General Systems*, vol. 28 (4-5), pp.351-366. In the Part II we deal with the development of the conceptual content of the Ternary Description Language and formalization in it the most important concepts of the Parametric General Systems Theory such as system descriptors and system parameters. Formal models of the 40 values of the binary attributive system's parameters are given.

Keywords: System's descriptor; system's parameter; attributive, relational, mereological, neutral implications; truth; contrary falsity; contradictory falsity; systems: conceptual point, structural point, rigid, totalitarian, minimal, immanent, centric, homeomery, elementary, unique, automodel, internal, homogeneous etc.

1. THE BRIEF EXPOSITION OF THE PART I

This section serves as an introduction to the Part II. It is a brief outline of what I have argued previously. First, I have showed the insufficiency of the existing mathematical and logical apparatus to express main concepts of *General Systems Theories* (GST). The formal apparatus appropriate for this purpose has to borrow some fundamental features from natural languages. One of them is *intensionality*, and the other is *incrementality*. (A formalism is called *incremental* in relation to a nonformal language if it is possible to increment the expressivity of the formalism with the help of that formalism itself.) Furthermore, our desirable formal language must be self-applicable just as natural one. (It was argued that this feature does not lead necessarily to paradoxes if special methods of language construction are used). Finally, it is desirable to broaden the sphere of the logical conclusions within the language by including not only deductions from judgments, but also deductions from terms.

Below follows the account of the essentials of logical formalism called *Ternary Description Language* (TDL) that pretends to be a language of Parametric GST and possesses features mentioned above. Firstly, it differs from traditional (Aristotelian) logic and also from Predicate logic in its categorial framework, which includes three categories: Things, Properties and Relations. (This accounts for the name TDL of our formalism). The peculiarities of our philosophical approach to these categories were described in the previous paper. Here it can be said that we do not reduce the difference between Property and Relation to the difference of one-placed and many-placed predicates. The essential feature of our conceptual framework is the contextual character of distinction between the categories. It means that a thing in one context can be a property or relation in another context. For instance, in the sentence “Love is a good affection” the word “Love” expresses a thing (= object = entity). In the sentence “That affection is love” the word “love” expresses a property. In the sentence “John loves Margaret” the word “loves” denotes a relation.

Things, properties and relations can be *definite*, *indefinite* and *arbitrary*. We denote the definite object by the symbol t, an indefinite object by the symbol a, an arbitrary object by the symbol A. Formulae

I. t, a, A

are elementary *well-formed formulae* (WFF) of our formalism – TDL. The other types of WFF are formed in the following manner:

II. $(A)A$

– Arbitrary thing (= object = entity) has an arbitrary property. Here we can substitute A for any WFF, and the result of such a substitution is considered as WFF too, e.g., $(A)a$, $(t)a$, $(a)t$, $((A)\,a)\,a$ are WFF. The same is valid for formulae given below.

III. $A(A)$

– Arbitrary thing has an arbitrary relation. $a(A)$, $a(t)$, $t(a)$, $a(a(A))$ are special cases of that type of WFF.

IV. $(A^*)A$

– This type of WFF differs from (II) in the direction of the predicate relation. The formula means that an arbitrary property belongs to an arbitrary thing, e.g.: $(a^*)A$, $(a^*)\,t$, $(a^*)\,a$, $(a^*)\,(a)\,t$.

V. $A(^*A)$

– An arbitrary relation realizes on an arbitrary thing, e.g.: $A(^*a)$, $t(^*a)$, $a(^*a)$, $t(a)(^*a)$.

The formulae of the types (II)-(III) may be called *direct* ones, and the formulae of the types (IV)-(V) – *inverse* formulae.

VI. $[A]$

– A formula of this type means the what may be called the *conceptual closure* of the formula A. If A expresses a proposition, then $[A]$ denotes the concept corresponding to that proposition. The conceptual closure of $(A)A$ gives us the formula $[(A)A]$ that is interpreted as "an arbitrary thing possessing an arbitrary property." Similarly $[(A^*)A]$ denotes "an arbitrary property inherent in the arbitrary thing", etc.

The formulae of the type (II)-(V) are *open*, while the formulae with square outermost brackets are *closed*.

VII. $\{A\}$

– Curly brackets have an ancillary character. They are used in the case when the inclusion of one formula into another as a subformula leads to ambiguity. E.g. $(A)\,a(A)$ may be understood as "A possesses the property $a(A)$", and also as "A possesses the relation $(A)\,a$". The first interpretation is expresses as $(A)\{\,a(A)\}$, the second as $\{(A)\,a\}(A)$.

VIII. A, A

– This type of WFF is a simple list of WFF. We shall call the formulae of such a type *free lists*, because they do not suppose any relation between their components. Note that the order of formulae in a list is ignored. The combinations of symbols t, a and a, t are regarded as one and the same combination.

Nevertheless, the order of symbols is very essential in the other types of WFF. We have seen it in the examples of direct and inverse formulae. The importance of that order gets its manifestation in the role symbol's place in a formula plays in its interpretation. The concrete meanings of A and a objects (but not t object) depend on their environment in a formula.

Let us explain this dependence. We distinguish the first – initial, and the second – final parts in every two-membered formula listed above. In direct formulae initial parts are included in parentheses. They denote things. In inverse formulae the initial parts are placed outside of parentheses. They denote properties and relations. In the open formulae, $\{(A)A\}$, $\{A(\ A)\}$, $\{(A^*)A\}$, $\{A(^*A)\}$, both A are completely arbitrary objects. In the closed formulae, A denotes the completely arbitrary object only if it is placed on their final parts. The arbitrariness of A on the initial part of a formula is *restricted*, e.g. in the $[(A)\,t]$ the symbol A denotes an arbitrary object that is restricted by the condition: "having the property t". An indefinite thing a, when it is placed on the initial part of an open formula, has an unlimited range of indefiniteness. However, when a appears in the final part of a formula, it has a restricted indefiniteness. In $\{(a)\,a\}$ the second a is an indefinite object, but is also a property of the first a. Correspondingly in $\{a(a)\}$ the first (that is final) a is a relation of the second (initial) a. In $\{(a^*)\,a\}$ the final a is a thing to which an initial a is prescribed as a property, and in $\{a(^*a)\}$ – as a relation.

In the case of complicated formulae, which consist of non-elementary subformulae, we can also (recursively) define the initial part, i.e. the *beginning* of the formula. The indefiniteness placed on the beginning of an open formula will be called *initial*, while the indefiniteness restricted by the sentence context – *contextual*. In the closed formulae the indefiniteness can be contextual even on the initial place, e.g. $[(a)t]$.

If there are two or more occurrences of the symbols a or A in the same formula, this does not mean, that they necessarily denote the same object. On the other hand, different subformulae can denote one and the same thing (just like in natural languages).

In those cases, when it is known that various occurrences of the same or of different subformulae denote *the same object*, this fact should be expressed with the aid of additional symbols in front of these subformulae. There is no need to include these symbols to the list of WFF's types since formulae with identifying symbols can be formally defined through WFF listed above, as it was shown in our previous paper. We have constructed our formal definition of identity based on well known principle that was formulated by Aristotle and is usually called Leibniz's principle: "That what is said about one thing should be said about the other".

Speaking about identity, we should take the direction of the identification into account. It is particularly important for us, because without the distinguishing of directions of identification we could not distinguish the operations of synthesis and analysis.

We use the small Latin letter j (italic jay) to denote an object with which the identification is being carried out: jA. Jay bold-faced letter in front of the formula denotes an object being identified: $\mathbf{j}A$. The assertion of the identity of any object to any object will have the form: jA jA. In particular: { ja ja }, { jA ja }.

Formulae with jay operators are analogous to the identity relation that is represented by “=” symbol, e.g. in algebra: $(a+b)^2 = a^2+2ab+b^2$. But in algebra we can observe other type of identifications, which are related to separate terms in formula. E.g., both occurrences of a in $a^2+2ab+b^2$ denote identical numbers. Algebra does not require a special symbol for that kind of identity because of the assumption that it is expressed by the identity of the forms of symbols. But in our case the same symbol a (or A) can denote different objects in different occurrences. Therefore if those denoted objects are factually identical, it is necessary to use special marks for the corresponding occurrences of terms.

In this paper we shall restrict ourselves to the case of the *undirected* identity of terms. We denote it by the Greek letter ι (iota) in front of formulae representing identified objects. It can be shown that iota operators can be formally defined through jay operators. Examples of iota operators usage: $(\iota A)\iota A$, $(\iota A^*)\iota A$. Not one, but many different identifications can occur in the same formula. In this case, several types of iota operators are used. In order to obtain the necessary variety of these operators the letter iota can have various subscripts, be doubled, tripled, etc. E.g.: $([(\iota A)\iota\iota A])[(\iota A)\iota\iota A]$, $([(\iota_8 A)\iota_{13}A])[(\iota_8 A)\iota_{13}A]$.

Let us now turn to problems of GST. Various authors give different definitions of the system’s concept. Some are too “broad”, e.g. “Systems are sets of objects with some relations between them”. We can express such a definition with the aid of the following TDL formula:

$$(\ \iota A\ )\mathrm{S}\ =_{def}\ \ a(\iota A) \tag{1.1}$$

Another kind of definition is connected to the specification of the "system-making" relation. Systems are defined as "Complexes of interacting elements", "Complexes of interconnected elements", or "Ordered sets of elements", etc. Such definitions can be expressed with the help of TDL formula:

$$(\iota A)\mathrm{S} =_{def} [(a)\, t\,](\iota A) \qquad (1.2)$$

All definitions of the form (1.2), where t is a concrete property, are too "narrow", because an attempt to find t that is appropriate for *any* system research fails. Every concrete t has its defects. From our point of view the solution of the problem is in the permission to *change the interpretation of* t. Instead of concrete t we will refer to t in general. In this case we can take the above formula not as a scheme of definitions but as a pure definition: "*A system is an arbitrary thing in which a relation having a definite property is realized*".

We can rewrite formula (1.2) in equivalent but more compact form:

$$(\iota A)\mathrm{S} =_{def} ([\, a(*\iota A)\,])\, t \qquad (1.3)$$

The majority of definitions given in the literature can be considered as particular cases of our definition (1.3). Nevertheless there are some exceptions. At times the role of definite t in the system definition moves to a relation (see Rapoport, 1966). In such cases we should have:

$$(\iota A)\mathrm{S} =_{def} (\iota A)[\, t\,(a)\,] \qquad (1.4)$$

$$(\iota A)\mathrm{S} =_{def} t(\,[(\iota A*)\, a]\,) \qquad (1.5)$$

In words: "*A system is an arbitrary thing in which properties having a definite relation between them are realized*". This definition is dual to the previous one in respect to transformation "property – relation."

In the following sections of the part II the concepts of system's descriptors and attributive system's parameters are analyzed. Several types of implications are introduced. With the help of implications the concept of Truth and (contrary and contradictory) Falsity are defined. In the final section the explications of values of several binary attributive parameters are given.

2. SYSTEM'S DESCRIPTORS AND ATTRIBUTIVE SYSTEM'S PARAMETERS.

Formulae (1.3), (1.5) of the previous section give evidence concerning that the system presentation of an object (i.e. construction of its system model) presupposes the separation of three aspects, which may be named *system descriptors*. First and foremost, *t* must be defined. Because *t* is immediately connected with the initial *conception* of the system, the term *concept* is prescribed to it. If the concept is a property as in the formula (1.3), then it is an *attributive* one. If the concept is a relation, as in the formula (1.5), then it is a *relational* one.

From this point on we shall restrict our consideration to the systems models with an attributive concept. Indicate the attributive concept with the symbol *P* (from Latin *proprietas* – property). A relation having the attributive concept as a property is called a *structure* (relational) and indicated with the symbol *R* (from Latin *relatio* – relation). A thing, on which a structure is realized, forms a *substratum* of systems. We indicate it with the letter *m* (from Latin *materia* – matter, substance, material). The concept, structure and substratum are the *system's descriptors of the first order*. They form the integral elements of a system model of any objects.

Note that, for any object *m*, there is always a concept *P* and such a structure *R*, which satisfies this concept, that will be realized by that object. Therefore in relation to those *P* and *R* a given object *m will be a system*. On the other hand, there are always such *P* and *R*, that a given object *m will not be a system* in relation to these *P* and *R*. This gives evidence on the *relativity* of the system's concept and the fact that systems *are not a species* of objects, but their *models,* which always can be constructed.

The aim of the system model construction consists in receiving a specific system information about an object. To reveal the weight or color of an object, it is not necessary to present this object as a system. It is another matter, if questions are posed about homogeneity or heterogeneity, or about the presence of a center, or about the stability or simplicity-complexity et cetera. For answers to these questions, it is necessary to present the object under investigation as a system.

The system information is determined by that class to which a system under consideration belongs. Thus we encounter the problem of the *systems classification*. In what way can that classification be constructed? The model of system, which was proposed above, gives the possibility to formalize the answer to that question.

Fundamenta divisionis in the systems classification are systems descriptors characteristics. Among those descriptors, which were defined above, only the concept has meaning in its own right. Its properties can be given without any relation to a structure and substratum. As for properties of the structure and substratum, they are determined only in relation to another system descriptors.

Relations between descriptors of the first order, i.e. the concept, structure and substratum, are *descriptors of the second order*. Their properties are essential as fundamenta divisionis of division of the general concept of system into classes. Indicate such descriptors. It is natural to express various modes of realization of the concept on a structure with the symbol P/R. The contrary relation – the structure to the concept is expressed as R/P. Relation of the structure to a substratum – R/m. It can be called the *structure organization*. Similarly – the relation of the substratum to the structure – *substratum organization* – m/R. On the basis of combinatory considerations it is possible to relate the concept P to the substratum m and vice versa – the substratum m to the concept P. However, such a relation has no practical interest, because the relation between the concept and the substratum always goes through the intermediary of the structure.

The order of the system's descriptors may be enhanced if we relate a descriptor of the second order to a descriptor of the first or the second order. In such a manner a system's descriptor of the third order would be obtained. However in practice such a descriptor and descriptors of higher order have seldom been used. In what follows we shall restrict ourselves to system's descriptors of the first and on the second order. Descriptors of a higher order will be taken into account only as a theoretical possibility.

The next scheme of the systems classes' definition may be concluded from the above:

$$(m)\text{System of a definite class} =_{def} ([R(*m)])P \bullet \\ \bullet \{(P)a \vee (P/R)a \vee (R/P)a \vee (R/m)a \vee (m/R)a\} \tag{2.1}$$

The meaning of the brackets here is the same as in TDL. a – the symbol of a property. The point denotes the sign of related list, joint acceptance, which is analogous to the conjunction. $\vee$ – the sign of disjunction, which is not included into the set of primitive symbols of TDL, but may be formally defined in its framework. The symbols of system descriptors are not formulae of TDL. Hence the whole formula (I) is not a formula of TDL. This is only a graphic demonstration, the purpose of which is an explication of the mechanism of construction of systems classes. A class of systems is determined when an indefinite property a receives its formal expression in a definite formula of TDL. It is evident that for all sets of descriptors it is possible to obtain an unlimited number (counting infinity) of TDL formulae, which express the properties of those descriptors. Therefore for each set of systems descriptors we shall receive an unlimited number of system classes in the same way.

Concrete formulae expressing peculiarities of system's descriptors determine those which can be named *values of attributive system's parameters*. Each of those values determines the corresponding class of systems.

The supplementary value of an attributive system's parameter determines the supplementary class of systems. All systems can be divided into two classes that are supplementary to one another, e.g. classes for having a center and not having a center, or homogeneous and heterogeneous systems, et cetera.

We call such attributive system's parameters that have only two supplementary to one another values *binary*. In some cases, determination of the two supplementary values of the attributive system parameters is inadequate, e.g. making evaluation of complexity or integrity of a system. Here it is necessary to determine the multitude of values, which are ordered in a linear way. We call such parameters *linear* ones.

The preceding means that an unlimited quantity of attributive system parameters may be correlated to every system's descriptor.

Any information received about a system can be expressed through characterizing it with values of attributive system's parameters. The purpose of a general systems theory is not only a description of systems in the language of this theory, but also to receive consequences of that description. M.Mesarovic and Y.Takahara are right when they note that "Study of logical consequences from the fact that systems have definite properties must be the basic content of any general systems theory" (Mesarovic, et.al., 1975).

There are two methods to obtain such results. The first is based on statistical relationships, which are established between values of attributive system's parameters. The parametric general systems theory has been developed in this way during the first stages of its existence. (Problems of the Formal Analysis of Systems, 1968; Portnov, et.al., 1972).

Combinations of values of attributive system's parameters on concrete objects were investigated. In this manner 31 general systems laws of statistical character were determined. E.g.: "There is no centric system that is not stationary" (Uyemov, 1978). On the basis of this law if we are confronted with a centric system, then we can suppose with high probability that it is stationary.

The statistical character of interconnections between values of system's parameters can give us only a probability of the obtained conclusion. Therefore in cases when a completely reliable conclusion is required, statistical methods are unsuitable. Here it is possible to use another, i.e. the analytical method of obtaining conclusions.

It may be used even if there is no axiomatic construction of the general systems theory. The difficulty of finding a general systems theory axioms was mentioned above. However such axioms turn out to be unnecessary if there is a formal apparatus with the help of which on the basis of formal models of some system's parameters values it is possible to determine formal models of another system's parameters values. TDL gives such a possibility. But in order to be able to express this, it is necessary to enrich its formalism so that it is possible to construct formal models of attributive system parameters values within its framework.

3. THE SUBSIDIARY CONCEPTS OF THE TERNARY DESCRIPTION LANGUAGE.

The most fundamental relations in a logical calculus are implications. They are usually defined with the aid of the truth function tables as a relationships between statements. Our position differs from the traditional one in two aspects. First of all, we are not assuming truth-falsity as primitive concepts. Analogues of truth and falsity will be defined below as derived concepts. Further, we recognize in our system the equivalence of sentences and concepts in principle (expressed by open and closed formulae respectively). Both sentences and concepts can act as antecedents and consequents of implications.

We shall make use of several types of implications. The first, called *attributive*, is defined with the aid of the following formula:

$$\{\ \iota A \Rightarrow \iota\iota A\ \} =_{def}\ \mathrm{j}\,\iota A\, j\ [(a)\,\iota\iota A] \qquad (3.1)$$

Here, the definiens[*] expresses identification of the object denoted by ιA with some object possessing properties expressed by $\iota\iota A$. This holds in categorical sentences expressed with the aid of the copulative verb “is”. When we say that a square is a rectangle, we mean that a square is identical to some object endowed with the properties of a rectangle.

The sequence of objects ιA and $\iota\iota A$ expressed as a formula of the list type $\iota A, \iota\iota A$ can be composed with various aims. If something is known about the objects ιA and $\iota\iota A$, then both these objects could be joined by a single pair of curly brackets merely for the sake of simplification. The list $\iota A, \iota\iota A$ will mean the same thing as ιA and $\iota\iota A$

[*] “Every definitional formula has two components, one containing the new sign and the other not. The component containing the new sign is called the definiendum; we shall follow the practice of writing the definiendum as the first, or left, component of the definition. The other component of a definition contains only earlier signs: it is called the definiens.” (R.Carnap, Introduction to Symbolic Logic and its Applications, Dover Publications, New York, 1958, p.64.) In the formula (3.1) the definiendum is $\{\iota A \Rightarrow \iota\iota A\ \}$ and the definiens is $\mathrm{j}\,\iota A\, j[(a)\,\iota\iota A]$.

separately. Instead of writing $a(*\iota A)$, $a(*\iota\iota A)$, we may write $a(*\{\iota A, \iota\iota A\})$. In this connection it is necessary to note, that here a relation a is not considered as a relation *between* objects ιA and $\iota\iota A$, but only as a relation in (or to) objects ιA and $\iota\iota A$ separately.

If objects of the list are considered separately, not relating to each other, then, even without denying the presence of a relation between them, we are dealing with an ordinary, simple *free list*.

If the list somehow relates objects to each other, then, in this case we shall call it *related* and denote it by $\{\iota A \bullet \iota\iota A\}$. We can give the following definition of such a list:

$$\{\ \iota A \bullet \iota\iota A\ \} =_{def} [\ (\iota\iota\iota A)\{[\ A(*\iota A,\ \iota\iota A)] \Rightarrow [a(*\iota\iota\iota A)]\}\ ] \tag{3.2}$$

The meaning of this definition resides in the fact that for a related list $\iota A \bullet \iota\iota A$ there is such an object $\iota\iota\iota A$ that any relation to an object ιA or $\iota\iota A$ will simultaneously be a relation to the object $\iota\iota\iota A$. Consequently, through a certain intermediate object – $\iota\iota\iota A$, the objects ιA and $\iota\iota A$ turn out to be related to each other.

Using the concept of a related list, let us define a new type of implication, which we shall denote by the symbol $\supset$:

$$\{\ \iota A \supset \iota\iota A\ \} =_{def}\ \mathrm{j}\iota A\, j\{\iota\iota A \bullet a\} \tag{3.3}$$

The implication defined by this formula is called *mereological*, since it corresponds to a generalized understanding of the relationship between a part and a whole, according to which a part may also be a subset and an element of a set. That relationship was studied by St.Lesniewski in his “Mereology” (Luschei, 1962).

The definiens of definition (3.3) means that a whole – ιA is identical to a part – $\iota\iota A$, to which something has been added. For example, Ukraine mereologically implies Odessa, because Ukraine is equal to Odessa with something else added.

The third type of implication is *relational*. It is analogous to the attributive implication, but the role of attribute in it is played by a relation. Let us denote the relational type of implication by the symbol $\succ$. We shall define it with the help of the next formula:

$$\{\ \iota A \succ \iota\iota A\ \} =_{def}\ \mathrm{j}\iota A\, j[\iota\iota A(a)] \tag{3.4}$$

E.g. a map of a city (ιA) implies relations between streets of that city ($\iota\iota A$). An object ιA is identical to some object with relations $\iota\iota A$.

We can generalize the three types of implications defined above in the concept of <u>neutral</u> implication. We use a simple arrow to denote it:

$$\{\ \iota A \to \iota\iota A\ \} =_{def}\ ([\,A(*\iota A \bullet \iota\iota A)])\{[(\iota A \Rightarrow \iota\iota A^*)\iota\iota\iota A],$$

$$[(\iota A \supset \iota\iota A^*)\iota\iota\iota A], [(\iota A \succ \iota\iota A^*)\iota\iota\iota A]\} \tag{3.5}$$

According to this definition, neutral implication is a relation between a list of related objects, which possesses an arbitrary property that can be ascribed to the attributive, mereological, and relational implications. For example, if it is shown that the attributive, mereological, and relational implications all possess the property of transitivity, then this property may be extended to the neutral implication. But if some property is inherent only to the attributive or only to the mereological or relational implication, then this is not sufficient to ascribe it to the neutral implication. At the same time, if a property is inherent to the neutral implication, then it is inherent to the attributive, mereological, and relational implications as well.

Note that in defining implications we did not make any use of valent values – Truth and Falsity. This grants us the possibility for inverting the problem, i.e. for posing the question of defining valent values through an implication introduced independently of them.

Truth and Falsity are regarded as some properties of TDL formulae, not only for open formulae but closed as well. In its turn, they have properties themselves. Truth has such properties as: a) It “carries over” from the antecedent of an implication (neutral, and hence, also every) to its consequent; b) It is preserved by repetition – truth of a true formula denotes a true formula.

Falsity has properties of the opposite nature: a) It “carries over” from the consequent of an implication to its antecedent; b) It is not preserved by repetition – falsity of a false formula denotes a true formula.

In as much as we define all those properties formally, we will obtain the formal definitions of a true and false formulae.

Let us denote the property of “carry over” from the antecedent of an implication to its consequent as a_c . Here the index *c* is from the word “consequent”. The formal definition of a_c will be:

$$a_c =_{def} \; [(\iota\, a)\{\; \{\; \iota\iota A \rightarrow \iota\iota\iota A \;\}\rightarrow \{[(\iota\iota A)\,\iota\, a] \rightarrow [(\iota\iota\iota A)\,\iota\, a]\}\;\}] \qquad (3.6)$$

Analogously, the opposite property of “carrying over” from the consequent to the antecedent – a_a (index *a* is from the word “antecedent”) will have the definition:

$$a_a =_{def} \; [(\iota\, a)\{\; \{\; \iota\iota A \rightarrow \iota\iota\iota A \;\}\rightarrow \{[(\iota\iota\iota A)\,\iota\, a] \rightarrow [(\iota\iota A)\,\iota\, a]\}\;\}] \qquad (3.7)$$

If a_c preserves itself by repetition, we denote it as a_{cp} (index *p* is from the word “preserve’) and express it formally in the following way:

$$a_{cp} =_{def} \; [(\iota\, a_c)\{\; \{([(\iota\iota A)\,\iota\, a_c])\;\iota\, a_c \;\} \Rightarrow \{(\iota\iota A)\,\iota\, a_c\}\;\}] \qquad (3.8)$$

In the opposite case we use the denotation a_{an} (index n is from “non”) and give its formal definition in such a way:

$$a_{an} =_{def} \; [(\iota\, a_a)\{\; \{([(\iota\iota A)\,\iota\, a_a])\;\iota\, a_a \;\} \Rightarrow \{(\iota\iota A)\, a_c\}\;\}] \qquad (3.9)$$

And finally we can express the property of returning of a_{cp} through the repetition of a_a. Denote it by a_{cpr} (index r is from "returning"):

$$a_{cpr} =_{def} \ [(\iota\, a_{cp})\{\ \{([(\iota\iota\iota A)\iota\iota\, a_a])\ \iota\iota\, a_a\ \} \Rightarrow \{(\iota\iota\iota A)\iota\, a_{cp}\}\ \}] \qquad (3.10)$$

Now we can define the "true" and "false" formulae:

$$(\iota\, A)\mathrm{T}\ =_{def}\ (\iota\, A)\, a_{cpr} \qquad (3.11)$$

$$(\iota\, A)\mathrm{F}\ =_{def}\ (\iota\, A)\, a_{an} \qquad (3.12)$$

Of course we do not pretend to have given definitions of Truth and Falsity in their broad philosophical connotations. We shall speak only of the construction of their formal, logical models.

Let us pay attention to the structure of the definitions (3.11-12) as well as all the previous definitions. They are built with the aid of A supplied with the iota operators. This gives us the possibility to formulate the next *rule of definitional deduction*:

If different occurrences of A in a definition are bounded by the same iota operator, then instead of all these occurrences *one may substitute an arbitrary (but anywhere the same) formula*. The iota operator may be *preserved* or *omitted* provided that single iota operators do not appear.

Thus our definitions can be transformed into an infinite number of various definitions. So, our definitions we can call *definition schemes*.

For example, let us substitute both occurrences of A in (3.11) for the formula $(A)a$. Instead of (3.11) we will obtain:

$$(\iota\{(A)a\})\mathrm{T}\ =_{def}\ (\iota\{(A)a\})\, a_{cpr} \qquad (3.13)$$

In the case of omitting of the iota operator we will have:

$$(\ (A)a\ )\mathrm{T}\ =_{def}\ (\ (A)a\ )\, a_{cpr} \qquad (3.14)$$

A subformula $\iota\, A$ in a formula $(\iota\, A)$T can, in its turn, be valent, i.e. ends with T or F or with other valent sign, which can be defined at a later time. Continuing further, we shall eventually arrive at a subformula which contains no valent signs. This will be the formula's *nucleus*. The collection of all valency signs which characterizes the nucleus, may be called the *valency ending*. If a formula is identical with its nucleus, i.e. has no valency ending, it may be called *nucleary*. Such is the subformula (A)a in the makeup of the formula ((A)a)T. Formulae may also be without any nucleus, i.e. *nucleariless*, e.g. ((T)a)T or ((T)T)T.

Nucleary formulae will be included in the class of the not-valent formulae. Not-valent formula may be used in two ways: as a nucleus of valent formula or as an independent one. Note that a not-valent formula, while not having valency, has some meaning, which is expressed by its structure. For example, the formula (a)A means that a thing has any property. It is true that the given formula has precisely this meaning. As not valent we can look at this formula for instance putting the question: is it a well-formed formula?

A not-valent formula can be defined with the help of another not-valent formula. In this case the definition expresses the identity of meanings of the two correlated formulae. The valent formulae were defined through not valent formulae in the definitions (3.11-12). It means that valent formulae are special cases of not-valent ones.

But, not all not-valent formulae are equivalent to some valent ones. Because of this we do not accept the so-called assertion principle, according to which the utterance of a statement is an assertion of its truthfulness (Carnap, 1946). Therefore, our notation for any not-valent formula, e.g. t or (t)a, does not mean that a formula, which has been written down, lays a claim to truth. t assumes truth if this formula will be the nucleus of the formula (t)T. Correspondingly, the truth of (t)a is assumed if there is ((t)a)T. The endings T or F can be used twice, three times, et seq. For example, it is possible to have ((((t)a)T)T)T or

$((((t)a)\mathrm{F})\mathrm{F})\mathrm{F}$. The first formula is equivalent to $((t)a)\mathrm{T}$, the second – to $((t)a)\mathrm{F}$.

There can be mixed endings containing different valency signs, for example $((A)\mathrm{F})\mathrm{T}$. The given formula is interpreted as follows: the assertion that "an arbitrary object is false" is true.

This should stir up protest on our part for the thought expressed here, that the entire world is an illusion. But how shall we express this protest? Apparently, we may write F in place of T, i.e. introduce a second negation. Hence $((A)\mathrm{F})\mathrm{F}$. However, in accordance with the definition of $(A)\mathrm{F}$, we shall get that the repeated falsity returns to the object's truth. Therefore, while $(A)\mathrm{F}$ signified the falsity of an arbitrary object, $((A)\mathrm{F})\mathrm{F}$ will denote the truth of an arbitrary object, i.e. $(A)\mathrm{T}$. Thus we have a recognition of the truth not only of the Pacific Ocean or of New York city but also of the present King of France, the naked King's clothes, the round square etc, because all these objects can substitute A in $(A)\mathrm{T}$.

What we have said indicates that falsity, in the form defined above, is not a fully adequate model of negation. Nevertheless, the more adequate model of it can be obtained on the basis of the valent formulae already defined.

Let us call the falsity, which we defined above, *contrary*. It can be applied directly to the nucleary formulae. *Contradictory* falsity (contradictory negation), which we will denote by the symbol n, can be applied only to the formulae which have already been valent. It can be defined with the help of the following formal definitions:

$$((A)\mathrm{T})n =_{def} (a)\mathrm{F} \qquad (3.15)$$

$$((A)\mathrm{F})n =_{def} (a)\mathrm{T} \qquad (3.16)$$

$$(((A)\iota A)\mathrm{T})n =_{def} ((a)\iota A)\mathrm{F} \qquad (3.17)$$

$$((\iota A (A))\mathrm{T})n =_{def} (\iota A (a))\mathrm{F} \qquad (3.18)$$

$$(((\iota A^*) A)\mathrm{T})n =_{def} ((\iota A^*)a)\mathrm{F} \qquad (3.19)$$

$$((A (^*\iota A))\mathrm{T})n =_{def} (a (^*\iota A))\mathrm{F} \qquad (3.20)$$

Instead of definitions (3.17-20) we may use only one expression:

$$((A \mathrm{R} \iota A)\mathrm{T})n =_{def} (a \mathrm{R} \iota A)\mathrm{F} \qquad (3.21)$$

Here R denotes the structure of a formula from the following list:

$(A)\iota A$, $\iota A(A)$, $(\iota A^*)A$, $A(^*\iota A)$, $[(A)\iota A]$, $[\iota A(A)]$, $[(\iota A^*)A]$, $[A(^*\iota A)]$.

This structure is the same on both sides of the definition scheme (3.21).

The usage of the definition scheme (3.21) allows us to add some more definitions to the definitions (3.17-20):

$$(([(A)\iota A])\mathrm{T})n =_{def} ([(a)\iota A])\mathrm{F} \qquad (3.22)$$

$$(([\iota A(A)])\mathrm{T})n =_{def} ([\iota A(a)])\mathrm{F} \qquad (3.23)$$

$$(([(\iota A^*)A])\mathrm{T})n =_{def} ([(\iota A^*)a])\mathrm{F} \qquad (3.24)$$

$$(([A(^*\iota A)])\mathrm{T})n =_{def} ([a(^*\iota A)])\mathrm{F} \qquad (3.25)$$

The scheme (3.21) can be generalized in the following formula:

$$((\iota\iota A \mathrm{Q} \{A \mathrm{R} \iota A\})\mathrm{T})n =_{def} (\iota\iota A \mathrm{Q} \{a \mathrm{R} \iota A\})\mathrm{F} \qquad (3.26)$$

Analogously we should have the next scheme:

$$((\iota\iota A \mathrm{Q} \{A \mathrm{R} \iota A\})\mathrm{F})n =_{def} (\iota\iota A \mathrm{Q} \{a \mathrm{R} \iota A\})\mathrm{T} \qquad (3.27)$$

Here Q denotes the structure of a formula from the list of 16 formulae:

$(A)\iota A$, $\iota A(A)$, $(\iota A^*)A$, $A(^*\iota A)$, $[(A)\iota A]$, $[\iota A(A)]$, $[(\iota A^*)A]$, $[A(^*\iota A)]$,
$(\iota A)A$, $A(\iota A)$, $(A^*)\iota A$, $\iota A(^*A)$, $[(\iota A)A]$, $[A(\iota A)]$, $[(A^*)\iota A]$, $[\iota A(^*A)]$.

As an example of the application of the definition schemes given above, let us consider the formula $((A \Rightarrow t)\mathrm{T})n$. Using the definition of the attributive implication, we obtain: $((\, \mathrm{j}A\ j[\,(a)t\,]\ )\mathrm{T}\ )n$. The next step should be the implementation of j-identity definition: $\mathrm{j}A\ j[\,(a)t\,] =_{def}$ $(A)[(\ [(A)[\ ([\,(a)t\,]^*)A\ ]\ ]^*)A]$. Let us now substitute ιA in the formula (3.17) for $[(A)\ [(\ [(A)[\ ([\,(a)t\,]^*)A\ ]\ ]^*)A]\ ]$ (iota operator in ιA can be omitted according to the rule of definitional deduction). We obtain:

$$(\ ((A)\ [(A)\,[(\,[(A)[\ ([(a)t]^*)A\,]\,]^*)A]\,]\ )\mathrm{T}\ )n =_{def}$$

$$(\ (a)\ [(A)\,[(\,[(A)[\ ([(a)t]^*)A\,]\,]^*)A]\,]\ )\mathrm{F}$$

And, returning to abbreviations: $(\ (\ A \Rightarrow t\ )\mathrm{T}\ )n =_{def} (\ a \Rightarrow t\ )\mathrm{F}$.

The contradictory negation can be applied in the opposite direction – from a to A. For this case, we have the following definitions:

$$((a)\mathrm{T})n =_{def} (A)\mathrm{F} \qquad (3.28)$$

$$((a)\mathrm{F})n =_{def} (A)\mathrm{T} \qquad (3.29)$$

$$(\ (a\ \mathrm{R}\ \iota A)\mathrm{T}\ )n =_{def} (A\ \mathrm{R}\ \iota A\ )\mathrm{F} \qquad (3.30)$$

$$(\ (\iota A\ \mathrm{R}\ a)\mathrm{T}\ )n =_{def} (\iota A\ \mathrm{R}\ A\ )\mathrm{F} \qquad (3.31)$$

$$((a\ \mathrm{R}\ \iota A)\mathrm{F}\ )n =_{def} (\ A\ \mathrm{R}\ \iota A\ )\mathrm{T} \qquad (3.32)$$

$$(\ (\iota A\ \mathrm{R}\ a)\mathrm{F}\ )n =_{def} (\iota A\ \mathrm{R}\ A\ )\mathrm{T} \qquad (3.33)$$

We can generalize all these definitions in two schemes:

$$((\ \iota\iota A\ \mathrm{Q}\ \{a\ \mathrm{R}\ \iota A\}\ )\mathrm{T}\ )n =_{def} (\iota\iota A\ \mathrm{Q}\ \{A\ \mathrm{R}\ \iota A\})\mathrm{F} \qquad (3.34)$$

$$((\ \iota\iota A\ \mathrm{Q}\ \{a\ \mathrm{R}\ \iota A\}\ )\mathrm{F}\ )n =_{def} (\iota\iota A\ \mathrm{Q}\ \{A\ \mathrm{R}\ \iota A\})\mathrm{T} \qquad (3.35)$$

Returning to our initial example, we can easily express our philosophical attitude with the aid of contradictory falsity. Instead of $(\ (A)\mathrm{F}\ )\mathrm{F}$ we must write $(\ (A)\mathrm{F}\ )n$.

Let us highlight one important point. The definitions and definition schemes given above show that the contradictory negation, as distinct from F, is not implemented in any formula. The sphere of the application of the definitions and schemes that contain n is restricted to the specific types of formulae, namely those which contain A or a without iota operators as subformulae.

Taking into account the contradictory falsity, we obtain the three types of valent endings of TDL formulae: T, F and n. Except that it is possible to introduce also the fourth valency, when a formula has the valencies T and F simultaneously. For example, it is true that (a)T, and also true that (a)F. In general form it can be expressed as: (a){T,F}. Formulae which have the values T and F simultaneously may be called *ambivalent* ones. It is possible to combine the ambivalence and contradictory negation. For example, if there is ((A)T)n and ((A)F)n simultaneously, we can express it in the united formula: ((A){T,F})n.

Valencies, which were considered above, may be called *definite* ones. *Indefinite valency* may be ascribed to non-valent formulae, if they can have valency in principle. In addition to this it can be defined *quasidefinite* valency, when one valency T or F is known and other is possible. A quasitrue formula is true or ambivalent one. Let denote such a valency by {T, }. Correspondingly quasifalsity is denoted as {F, }. Quasitruth does not contradict to truth, and quasifalsity does not contradict to falsity of the corresponding formulae.

Let us return to the assertion principle. It can be expressed in such a form:

$$(\iota A \Rightarrow (\iota A)\mathrm{T}\,)\mathrm{T} \qquad (3.36)$$

We contrarily negate it. Therefore there is:

$$((\iota A \Rightarrow (\iota A)\mathrm{T}\,)\mathrm{T}\,)\mathrm{F} \qquad (3.37)$$

Note that the application of the assertion principle in classical logic has two limitations. First, it is applicable only to statements, which we express by the open formulae. In classical logic only statements can be evaluated as true ones. Second, it is applied only to a formula as a whole, not to separate parts of it, i.e. subformulae. In the statement calculus the utterance $a \to b$ may be regarded as true, but its components *a* and *b* are not regarded so. Their valency is unknown. If we know that *a* is true, we must write it separately: $a \to b$, a .

In our system closed formulae can be valent just as open ones. Similarly, subformulae can be valent or not. For example, in (3.36) the subformula ιA, which denotes the consequent, is valent, just as the formula of implication in its entirety. But ιA in the antecedent may be not valent.

The abandonment of the assertion principle leads to the usage of a large number of valency signs, which would make formulae too unwieldy. In order to avoid it, let us consider conventions for simplifications:

a) We will consider as not-valent such a formula, where the valency is unknown. Valency markers are determined on the basis of axioms and rules of inference, so these markers may be omitted in the case when their determinations are trivial. For example, in the formula (t,a,a,t)T there is no necessity to mark valencies of the every subformulae. According to the rule for lists RC_1 all of them are true.

b) If the valency of the whole formula does not depend on valencies of subformulae, those subformulae may be regarded as not-valent. E.g., the formula

$$((a)\{\mathrm{T,F}\}\,)[(\,(A)\{\mathrm{T,F}\}\,)n] \qquad (3.38)$$

has the valency F regardless of subformulae valencies. Therefore we may have the simpler formula $(a)A$ instead of (3.38).

c) It is supposed that the definiens and definiendum are true simultaneously. Therefore if the definiendum is true, the definiens is true also. In this case the valent sign T may be omitted both after the definiendum and definiens.

d) Because in this paper implications of any kind will be seldom used as not-valent formulae, and on the contrary, true implications will be used very often, it is convenient to agree here that the absence of valency sign after an implication denotes the mark T. If an implication lacks the valency T, we will always follow it with one of these symbols: F, $\mathrm{T})n$, $\mathrm{F})n$, {T,F} , {T, } , {F, } . If an implication must be considered as a not-valent formula, it will be denoted by inverted commas, e.g. "$A \Rightarrow a$".

To formalize the system parameters values we need to define some specific "objects" by application of previously defined operations to the basic objects a, A, t of TDL.

Let t' be an indefinite object which is different from t. Formally:

$$t' =_{def} [(\iota a)\{ (\{ \iota a \supset t \} \bullet \{ t \supset \iota a \})\mathrm{F} \}] \qquad (3.39)$$

In accordance with the accepted notations, the definiens of (3.39) means that it is impossible to have both implications simultaneously. And this impossibility is the property of ιa.

An indefinite subobject – $\overset{\cup}{t}$ – has a definition:

$$\overset{\cup}{t} =_{def} [(\iota t')\{ t \supset \iota t' \}] \qquad (3.40)$$

This is some object, different from t, possesses the property that t contains it. In some cases such an object corresponds to the notion of a *physical* or *extensional* part of the object *t*. E.g., a man contains his head as a physical part. A different kind of a part is an *intensional* one. In this case it may be a collection of properties. A man "contains" an animal in the sense that he possesses properties of an animal, so the collection of these properties is a part of a man. This part may be called *qualitative* or *attributive* one. An intensional part may represent also some collection of relations in the object *t*. In this case we may call it *relational* one. E.g., a city contains streets as relations between buildings. Buildings themselves are extensional parts of a city.

An indefinite superobject – $\overset{\Delta}{t}$ – is defined as:

$$\overset{\Delta}{t} =_{def} [(\iota t')\{ \iota t' \supset t \}] \qquad (3.41)$$

Unlike the preceding (3.40) here the question is not about *t* containing an indefinite object $\iota t'$, but of t being contained in $\iota t'$. In extensional sense t' may be e.g. a family, in which a man t is included. In intensional sense t' may be t in some special state, e.g. t' is a young man, where t is simply a man. It is an *attributive* superobject. The object *t*, to which a some relation is added, is *relational* superobject. E.g. Romeo and Julia in love with one another construct such a relational superobject different to that couple without love.

An indefinite disparate – $\overset{\circ}{t}$ – is defined as an object, different from *t*, which possesses two properties simultaneously: it is neither a subobject nor a superobject. Formally:

$$\overset{\circ}{t} =_{def} [(\iota t')\{ \{((\iota t'^{*})\overset{\cup}{t})F\} \bullet \{((\iota t'^{*})\overset{\Delta}{t})F\} \}] \qquad (3.42)$$

Finally, we will give the definition of the *limited* definite object. The idea of such an object is the following. The addition of something to a definite object does not usually mean that the entity has become different. For example, if we put gloves on the cat in boots it will still remain the cat in boots, in spite of the fact that gloves are not a part of boots. The situation is different if we have a cat *in boots only,* as should be supposed according to Perro's well-known tale. In order to emphasize this point, we should say that every addition that preserves a definite entity t (i.e. $\{t \bullet \iota A\} \Rightarrow t$) must occur in it (i.e. $t \supset \iota A$). The latter is presupposed by the former and is an element of the former. A definite object possessing the indicated property we denote as *Lt* and call it "the limited object".

$$Lt =_{def} [(\,t\,)\{ \{ \{ t \bullet \iota A \} \Rightarrow t \} \supset \{ t \supset \iota A \} \}] \qquad (3.43)$$

All the definitions that were given above may mutatis mutandis be applied to the iotafied object:

$$\iota A' =_{def} [(\iota\iota a)\{ (\{ \iota\iota a \supset \iota A \} \bullet \{ \iota A \supset \iota\iota a \})\mathrm{F} \}] \qquad (3.44)$$

$$\iota \overset{\cup}{A} =_{def} [(\iota A')\{ \iota A \supset \iota A' \}] \qquad (3.45)$$

$$\iota \overset{\Delta}{A} =_{def} [(\iota A')\{ \iota A' \supset \iota A \}] \qquad (3.46)$$

$$\iota \mathring{A} =_{def} [(\iota A')\{ \{((\iota A'^{*})\,\iota \overset{\cup}{A}\,)\mathrm{F}\} \bullet \{((\iota A'^{*})\,\iota \overset{\Delta}{A}\,)\mathrm{F}\} \}] \qquad (3.47)$$

$$L\iota A =_{def} [(\iota A\,)\{ \{ \{ \iota A \bullet \iota\iota A \} \Rightarrow \iota A \} \supset \{ \iota A \supset \iota\iota A \} \}] \qquad (3.48)$$

Let us finish this section of our article with the two definitions. First is the definition of two kinds of disjunction:

$$\{\iota A \vee \iota\iota A \} =_{def} \{ \{(\iota A)\mathrm{F} \rightarrow (\iota\iota A)\mathrm{T} \} \bullet \{(\iota\iota A)\mathrm{F} \rightarrow (\iota A)\mathrm{T} \} \} \qquad (3.49)$$

$$\{\iota A \;\mathrm{W}\; \iota\iota A \} =_{def} \{ \{\iota A \vee \iota\iota A \} \bullet \{((\iota A)\mathrm{T} \bullet (\iota\iota A)\mathrm{T})\mathrm{F}\} \} \qquad (3.50)$$

Second is the definition of *interjective* relation. It is expressed with the word "inter" (between). A relation exists *between* components of a related list, if it exists in that related list, but does not exist in components of that list taken separately. E.g., the relation "husband" exists in a couple (Peter, Mary), but it does not exist in Peter and in Mary taken separately. Formally interjective relation may be expressed in the following definition:

$$\iota A_{inter}(\iota\iota A \bullet \iota\iota\iota A) =_{def} \{ (\iota A(\iota\iota A \bullet \iota\iota\iota A))T \} \bullet \{ (\iota A(\iota\iota A, \iota\iota\iota A))F \} \qquad (3.51)$$

The index *inter* may be reduced to *int* and even to *i* .

4. THE VALUES OF BINARY ATTRIBUTIVE SYSTEM'S PARAMETERS AND THEIR FORMALIZATION IN THE TDL FRAMEWORKS.

Let's return to the scheme (2.1). Consider the first member of our disjunction (*P*)*a* which determines those characteristics of the concept which can be used to single out the value of a binary attributive system's parameter. The concept *P* is a property, *a* – some property of *P*. Therefore the classification of properties is supposed.

In the framework of such a classification let us use the distinction between the simple definite object *t* and the limited definite object *Lt* from the previous section. The property that is associated with the limited definite object may be called *a point property*. It doesn't allow for any variations. Aristotle gave us an example of such a property – "triangular" (*Categories*). This can be distinguished from the property "white", which permits some variations. This is an example of "*a non-point property*."

All the systems can be divided in two big classes according to these properties. The first includes those systems, which have point properties as concepts. The second includes those systems, which have non-point concepts. We call the first – *conceptual-point systems*, the second – *conceptual-non-point* ones.

As an example of the conceptual-point system one may take the natural series of numbers, the concept of which cannot be more or less. The relations of the type: “more clever”, “more beautiful” do not satisfy the concept of the full or strict ordering. Here only partial ordering may exist, which permits some deviations from the strict order.

The value of the system’s parameter “conceptual-point” may be formalized in such a way:

$$(\iota A)\text{Conceptual-point system} =_{def} (\iota A)\{([a(*\iota A)])Lt\} \qquad (4.1)$$

The complementary class of systems is defined so:

$$(\iota A)\text{Conceptual-non-point system} =_{def}$$
$$(\iota A)\{\{([a(*\iota A)])t\}\bullet(t \Rightarrow Lt)\mathrm{F}\} \qquad (4.2)$$

These definitions belong to the class of *explications*. Here definiendums are not formulae of TDL. They include some extralogical expressions, e.g. “system”, “conceptual-point”. Definitions do not make them formulae of TDL. They give only formal equivalents of those expressions. As distinct from explications, other kinds of definitions represent an introduction of denotations, with the help of which a simple symbol in definiendum is equated to more complicated one in definiens. Definitions (3.39-3.48) are definitions of such a type. Definiendums of those definitions became formulae of TDL with meanings taken from their definienses.

In both cases of definitions definiendums and definienses may be as nucleary, i.e. non-valent, as valent formulae. Non-valent formulae can be made valent ones by putting of the mark T after the definiendum and definiens simultaneously. According to the convention c) from the previous section, the mark T may be omitted after the definiendum and definiens simultaneously.

There exist not only conceptual-point systems, but also *structural-point systems*. A structural-point system is a system, the structure of which is uniquely determined, without any variations. To put it otherwise, the system’s structure must be a *limited object*. At the same time the structure need not to be a definite object.

The definition of a limited indefinite object is possible to obtain by substitution of *A* for *a* in the definition scheme (3.48). We will have:

$$L\iota a =_{def} [(\iota a)\{ \{ \{ \iota a \bullet \iota\iota a \} \Rightarrow \iota a \} \supset \{ \iota a \supset \iota\iota a \} \}] \qquad (4.3)$$

Changing iota-operators, we will obtain the next definition:

$$(\iota A)\text{Structural-point system} =_{def} (\iota A)\{([L\iota\iota a (*\iota A)])t\} \qquad (4.4)$$

In definiens here you see a single iota operator $\iota\iota a$. However, there are paired iota operators in the definiens of $L\iota\iota a$ definition.

The complementary class of structural-non-point systems is defined below:

$$(\iota A)\text{Structural-non-point system} =_{def}$$
$$(\iota A)\{ \{([\iota\iota a (*\iota A)])t\} \bullet (\iota\iota a \Rightarrow L\iota\iota a)\text{F} \} \qquad (4.5)$$

In definitions (4.4-5) we have used the convention c) about omitting the mark T after the definiendum and definiens of definition. We shall continue to use this convention.

While the division of systems according to the distinction between "conceptual-point system – conceptual-non-point system" pertains to the concept (taken as a system's descriptor); the system's parameter with values "structural-point/non-point system" supposes another system's descriptor – the relation of the structure to the concept – *R*/*P*. Another parameter which pertains to that descriptor is a fundamentum divisionis of systems into *structurally open* and *structurally closed* ones. In the first case, the structure permits its complication without going out of the frameworks of the concept *t*, i.e. changing into another system. The other situation would take place in a structurally closed system. Here any complication of the structure demolishes the system. Plato supposed that a master and his slaves constitutes a structurally closed system, which is founded solely on the relation of domination – submission. Another view was Aristotle's who admitted a friendly relation between a master and his slaves. Therefore Aristotle regarded the system "Master – slaves" as structurally open (Aristotle, *Politics*).

The next formal definitions might be assumed:

(ιA)Structurally open system $=_{def}$

$$(\iota A)\{\ \{(\ [\iota\iota a\ (*\iota A)]\ )t\}\ \bullet\ \{(\ [\iota\iota \overset{\Delta}{a}(*\iota A)]\ )t\}\} \qquad (4.6)$$

(ιA)Structurally closed system $=_{def}$

$$(\iota A)\{\{(\ [\iota\iota a\ (*\iota A)]\ )t\}\bullet\{(\ (\ [\iota\iota \overset{\Delta}{a}(*\iota A)]\ )t\ )F\}\} \qquad (4.7)$$

The next parameter that reflects other relation of a structure to a concept is "*structural variability* / *non-variability*". In structurally non-variable systems there are no relations that are different from the system-forming ones. As in the examples of the natural series of numbers or the abstract notion of a triangle. A structurally variable system has some relations that are different from the structure of a system. So, in a student group there are relations – of friendship, hostility etc., – that are different from group-forming ones. Let us give the formal definitions:

(ιA)Structurally non-variable system $=_{def}$

$$(\iota A)\{\{([\iota\iota a\,(*\iota A)])t\}\bullet\{[A(*\iota A)] \Rightarrow \iota\iota a\}\} \qquad (4.8)$$

(ιA)Structurally variable system $=_{def}$

$$(\iota A)\{\{([\iota\iota a(*\iota A)])t\}\bullet\{([A(*\iota A)] \Rightarrow \iota\iota a)n\}\} \qquad (4.9)$$

Note that in this definition the contradictory negation is used. It negates the truth of an attributive implication. By doing so, it asserts that for some relations in the system this implication is false, i.e. there is $([a(*\iota A)]\Rightarrow\iota\iota a)$F. This is not contradict the case where for some other relations that implication is true, i.e. $([a(*\iota A)]\Rightarrow\iota\iota a)$T also takes place.

Let us now change the direction of system descriptor *R*/*P*. We receive *P*/*R*, i.e. the relation of the concept to a structure. According to this descriptor it is possible to single out a new parameter. Let us consider a system, where the structure is uniquely determined by the concept. Such systems should be called *rigid*. For example of a rigid system, consider a problem which has only one feasible solution. Formally, it can be expressed as:

(ιA)Rigid system $=_{def}$ $(\iota A)\{\{([\iota\iota a\,(*\iota A)])t\}\bullet\{t \rightarrow \iota\iota a\}\}$ (4.10)

In the other cases the structure is not uniquely determined by the concept. The concept can be realized by various structures. This occurs in such problems that can be solved by various methods. Such systems should be called *non-rigid*. It is possible to give them the next formal definition:

(ιA)Non-rigid system $=_{def}$

$$(\iota A)\{\{([\iota\iota a\,(*\iota A)])t\}\bullet\{(t \rightarrow \iota\iota a)\mathrm{F}\}\} \qquad (4.11)$$

Another parameter, which is distinguished according to the descriptor *P*/*R*, has as its first value the kind of systems, which was the ideal of every totalitarian state, e.g. "*State*" by Plato, "*1984*" by Orwell or the non perfect realization of such ideas in the empire under Stalin, the third Reich under Hitler, China under Mao, North Korea, Kampuchea etc. This ideal holds that any relation in a state must be determined by its goal – the concept of a system. Such type of systems should be called *totalitarian* ones. Its formal definition is:

(ιA)Totalitarian system $=_{def}$

$$(\iota A)\{\{([a(*\iota A)])t\}\bullet\{t \rightarrow [A(*\iota A)]\}\} \qquad (4.12)$$

Systems of that type do not exist in society only. A good example of such a system is the natural series of numbers. All relations in this system are determined by its concept – the full order.

The complementary class – "non-totalitarian systems" may be obtained by the negation of the second set of braces in the definiens of the "totalitarian systems" definition. In negating the second set of braces we do not negate the possibility that some relation in the substratum could be the consequence of the concept. This means that the negation must have a contradictory, non-contrary character. Therefore we have the next formal definition:

(ιA)Non-totalitarian system $=_{def}$

$$(\iota A)\{\{([a(*\iota A)])t\}\bullet\{(t \rightarrow [A(*\iota A)])n\}\} \qquad (4.13)$$

Many system’s parameters correspond to the system’s descriptor R/m, i.e. they determine the type of relations of the structure to the substratum. Let’s begin the consideration of those parameters with the division of systems between *non-minimal* and *minimal*.

Non-minimal systems admit a removal of some of their elements without a destruction of the system as a whole. E.g. troops at war can suffer losses without ceasing to function as a system. Formally it can be defined as:

(ιA)Non-minimal system $=_{def}$

$$(\iota A)\{\{(\,[\iota\iota\, a(*\iota A)]\,)t\}\ \bullet\{(\,[\iota\iota\, a(*\iota \overset{\cup}{A})]\,)t\}\} \tag{4.14}$$

There are also systems, which do not admit the removal of elements without the destruction of the system as a whole. E.g. a quadrangle without an angle is not a quadrangle at all. Such systems we call *minimal* ones. Give the next definition:

(ιA)Minimal system $=_{def}$

$$(\iota A)\{\{(\,[\iota\iota\, a(*\iota A)]\,)t\}\bullet\{((\,[\iota\iota\, a(*\iota \overset{\cup}{A})]\,)t)\mathrm{F}\}\} \tag{4.15}$$

Here we use an usual or contrary negation because there is no arbitrary object in the negated formula.

Another system’s parameter is connected with the question: does a system-forming relation embrace only the system substratum elements or it embraces some elements outside this substratum? The movement of our troops forms a system, and it may corresponds strongly with the movement of hostile troops. However the movement of hostile troops is not included in the substratum of our system. The movement of troops on the parade is a different matter. Here a system-forming relation embraces only the substratum of a system. Naturally, in this case it is also necessary to take the conditions of the surroundings into account. This account has a different character from the account of a movement of hostile troops. Unsuitable conditions can spoil a parade, but they do not determine its structure.

We will call *immanent* a system, where the structure is not beyond its substrata. The complementary class is *non-immanent* systems. The formal definitions:

$$(\iota A)\text{Non-immanent system} =_{def} (\iota A)\{([a(*\iota A \cdot \iota \mathring{A})])t\} \qquad (4.16)$$

$$(\iota A)\text{Immanent system} =_{def}$$
$$(\iota A)\{\{([\iota\iota\, a(*\iota A)])t\} \cdot \{([\iota\iota\, a(*\iota A \cdot [(A)\iota\mathring{A}])])\mathrm{F}\}\} \qquad (4.17)$$

Bear in mind that symbol $\iota\mathring{A}$ denotes disparate of the substratum ιA. The contrary falsity (F) means that the ascription of the structure $\iota\iota\, a$ to the synthesis of the substratum ιA with any disparate $[(A)\iota\mathring{A}]$ is false.

Note that the change of a value of the considered system's parameter may lead to fundamental discoveries. So – from Newton's point of view – time flows “sine relatione ad externum quodvis”, i.e. represents an immanent system. As for Einstein, he represented time as non-immanent system relating time flow to movement of matter.

The next system's parameter is *centricity*. There may be an element among all the elements of a system, such that relations between any of the elements of the system can be established only through the relations to this (central) element. Those systems can be named *internal-centric*. As an example take a “man-machine” system, in the case when interactions between the elements of the machine can be realized only with the help of a man, i.e. the machine is not an automaton.

If a machine is considered separately from a man who controls it, it forms a system of *external-centric* type because the center is outside the system.

The notion that generalizes internal and external centricity is simply *centricity*. The complementary (to centricity) value of the parameter is *non-centricity*. Give the next formal definition:

(ιA)Internal-centric system $=_{def}$

$$(\iota A)\{\{([a(*\iota A)])t\} \bullet\{\iota\iota\,\{\iota\overset{\cup}{A}\} \bullet\{[A(*\iota A)] \Rightarrow [a(*\iota\iota\,\{\iota\overset{\cup}{A}\})]\}\}\} \quad (4.18)$$

The second set of braces means that an arbitrary relation of the substratum is some relation which is realized on a subobject of the substratum – the center – $\iota\iota\,\{\iota\overset{\cup}{A}\}$.

(ιA)External-centric system $=_{def}$

$$(\iota A)\{\{([a(*\iota A\bullet\iota\mathring{A})])t\} \bullet\{\iota\iota\{\iota\mathring{A}\} \bullet\{[A(*\iota A)] \Rightarrow [a(*\iota\iota\{\iota\mathring{A}\})]\}\}\} \quad (4.19)$$

Here the first set of braces means that system-forming relation embraces not only the substratum ιA but its disparate $\iota\mathring{A}$ too. The second set of braces indicates that an arbitrary relation in the substratum is a relation to some of its disparate – the center – $\iota\iota\,\{\iota\mathring{A}\}$.

(ιA)Centric system $=_{def}$

$$(\iota A)\{\{([a(*\iota A)])t\} \bullet \{\iota\iota a \bullet\{[A(*\iota A)] \Rightarrow [a(*\iota\iota\, a)]\}\}\} \quad (4.20)$$

Here an internal center $\iota\iota\,\{\iota\overset{\cup}{A}\}$ or external center $\iota\iota\,\{\iota\mathring{A}\}$ is replaced by the simple center – $\iota\iota a$.

(ιA)Non-centric system $=_{def}$

$$(\iota A)\{\{([a(*\iota A)])t\} \bullet\{(\iota\iota A \bullet\{[A(*\iota A)] \Rightarrow [a(*\iota\iota\, A)]\})\mathrm{F}\}\} \quad (4.21)$$

Here in the second set of braces there is a contrary negation which means the impossibility to be a center for an arbitrary object.

The parameter *homeomerity* was examined in ancient times. The term “homeomeria” (ta homoimere) is defined by Aristotle. By this term he meant things, in which the structure of the parts is the same as the structure of the whole. Examples are: copper, gold, silver, tin, iron, stone, etc., and also in an animal and plant: flesh, bones, tendons, skin, etc. (Aristotle, *Meteorology*).

We can say with reasonable confidence that Aristotle denoted the value of the system parameter, which we call *homeomerity*. The definition is:

(ιA)Homeomery system $=_{def}$

$$(\iota A)\{\ \{(\ [\iota\iota\, a(*\iota A)]\ )t\} \bullet \{(\ [\iota\iota\, a(*[(A)\iota\overset{\cup}{A}\ ]\ )t\}\ \} \qquad (4.22)$$

The second set of braces in the definiens means that the structure immediately relates to any subsystem (part) of the defined system.

The negation of homeomerity may be defined so:

(ιA)Non-homeomery system $=_{def}$

$$(\iota A)\{\ \{(\ [\iota\iota\, a(*\iota A)]\ )t\ \} \bullet \{((\ [\iota\iota\, a(*[(A)\iota\overset{\cup}{A}\ ]\ )t)\mathrm{T})n\}\ \} \qquad (4.23)$$

The contradictory character of the negation represents the fact that in a non-homeomery system there may be some parts which are similar to the whole.

Further, all systems may be divided into *elementary* and *non-elementary* ones. Any element of a system may be considered individually as a system. There may be two cases. In the first, none of the elements of the system is a system in the same sense as the whole. In other words, the individual elements are not systems with the same concept as the initial system. Such systems we call *elementary*. E.g. a family is an elementary system because none of its elements is a system in the same sense in which the family is a system.

In a *non-elementary* system there are elements which are systems with the same concept that the whole system has. E.g. the Solar system is a system of that kind because there are such subsystems in it – Jupiter, Saturn with their satellites – which themselves are similar to the Solar system. Formally the distinction between the non-elementary and elementary types of systems can be expressed with the help of the next definitions:

(ιA)Non-elementary system $=_{def}$

$$(\iota A)\{\{(\,[\,a(*\iota A)]\,)t\}\bullet\{(\,[\,a(*\iota \overset{\cup}{A})]\,)t\,\}\} \tag{4.24}$$

Note the absence of iota operators applied to the structure in the definition. That indicates the possibility of various structures of the initial system and those, which exist in substratum elements. Here only the identity of concepts is essential.

(ιA)Elementary system $=_{def}$

$$(\iota A)\{\{(\,[a(*\iota A)]\,)t\,\}\bullet\{(((\,[a(*\iota \overset{\cup}{A})]\,)t\,)\mathrm{T})n\}\} \tag{4.25}$$

Using the definition of the contradictory negation, the definiens of that definition can be expressed in the next form:

$$(\iota A)\{\{((\,[a(*\iota A)]\,)t)\mathrm{T}\}\bullet\{((\,[\,A(*\iota \overset{\cup}{A})]\,)t\,)\mathrm{F}\}\} \tag{4.26}$$

Further, systems may be *unique* and *non-unique*. In unique systems a system-forming relation can't be realized on any object different from the substratum. Works of famous artists are unique systems. Any reconstruction of their structures on other substrata, no matter how precise, is considered as a forgery. On the other hand, polygraphic copies of those works are non-unique systems, because they are realized on various substrata. As for other examples, the problem of uniqueness of the human personality is very interesting.

(ιA)Non-unique system $=_{def}$

$$(\iota A)\{\{([\iota\iota a(*\iota A)])t\} \bullet \{([\iota\iota a(*\iota A')])t\}\} \qquad (4.27)$$

(ιA)Unique system $=_{def}$

$$(\iota A)\{\{([\iota\iota a(*\iota A)])t\} \bullet \{(([\iota\iota a(*\iota A')])t)\mathrm{F}\}\} \qquad (4.28)$$

Now let us analyze *substratum-open* and *-closed* systems. Structurally open and closed systems were considered above. They belong to the system's descriptor *R*/*P*, i.e. the relation of the structure to the concept. Substratum openness and closeness belong to the system's descriptor *R*/*m*, i.e. the relation of the structure to the substratum.

Substratum-open systems permit the addition of new elements to substratum without changing the system's character. E.g. an edition of a book may be updated, new students may come during the lecture, etc. However adding a new angle to a quadrangle destroys that quadrangle: it is a substratum-closed system.

(ιA) Substratum-open system $=_{def}$

$$(\iota A)\{\{([\iota\iota a(*\iota A)])t\} \bullet \{([\iota\iota a(*\iota \overset{\Delta}{A})])t\}\} \qquad (4.29)$$

(ιA)Substratum-closed system $=_{def}$

$$(\iota A)\{\{(\,[\iota\iota a(*\iota A)]\,)t\,\}\bullet\{((\,[\iota\iota a(*\iota\overset{\Delta}{A})]\,)t\,)\mathrm{F}\}\} \qquad (4.30)$$

Here we have finished the consideration of system's parameters which are connected with the system's descriptor R/m. Now we shall analyze some parameters, which are connected with the inverse meaning of that descriptor, i.e. with the relation of the substratum to the structure – m/R.

Consider the parameter *automodelity*. Automodel systems are such systems, in which every element has properties of the system as a whole. Therefore the system models itself in everyone of its elements. Let us take "Material thing" as a system. Material thing consists of material things, i.e. elements of a material thing are material things. Any properties of the system as a whole refer to anyone of its elements.

Note the difference between automodel and homeomery systems. The price of a pound of gold is much more then the price of a gram. Hence homeomery system "a pound of gold" is not automodel.

Give the formal definition:

(ιA) Automodel system $=_{def}$ $(\iota A)\{_3\,\{_1\,(\,[\iota\iota a(*\iota A)]\,)t\,\}_1\,\bullet$

$$\bullet\{_2(\,[(A)\iota\overset{\cup}{A}\,]*)[([(\iota A)\{_1[\iota\iota a(*\iota A)]\,)t\}_1]*)A]\,\}_2\}_3 \qquad (4.31)$$

(ιA) Non-automodel system $=_{def}$ $(\iota A)\{_3\,\{_1\,(\,[\iota\iota a(*\iota A)]\,)t\,\}_1\,\bullet$

$$\bullet\{_2(((\,[(A)\iota\overset{\cup}{A}\,]*)[([(\iota A)\{_1[\iota\iota a(*\iota A)]\,)t\}_1]*)A]\,\}_2)\mathrm{T})n\}_3 \qquad (4.32)$$

Internal and *external* systems. If there are numbers, e.g. 4 and 5, then there is the relation between them: 4<5 and it is impossible to be otherwise. We call the relation “<” an *internal* relation between numbers. But if there are John and Peter, then we do not know who is a superior. “To be a superior” is an external relation between people.

The question about existence of external relations was called “the Great Question of Philosophy” (W.James). For us, as well as for James, there is no doubt about the positive solution of that question. We use it in the division of all systems into *internal*, the structure of which are internal relations, and *external*, the structure of which includes external relations. For the formalization of this distinction let us use the relational implication $\succ$ that was defined in (3.4):

$$(\iota A)\text{ Internal system} =_{def} (\iota A)\{\{([\iota\iota a(*\iota A)])t\}\bullet\{\iota A \succ \iota\iota a\}\} \qquad (4.33)$$

$$(\iota A)\text{External system} =_{def} (\iota A)\{\{([\iota\iota a(*\iota A)])t\}\bullet\{(\iota A \succ \iota\iota a)\mathrm{F}\}\} \qquad (4.34)$$

As distinct from simply internal systems there are *elementary internal* ones. If in the first case the structure is relationally implicated by the substratum as a whole and nothing is said about that implication for each one of its elements, then in second case it is supposed that the structure is implied by each element of the system. E.g. a collection of tractor’s details is an internal system. It is known that three points determines a circle. Therefore any three points of a circle implicates the circle as a whole. Thus a system of three points of the circle is elementary internal. On the contrary, a system, elements of which consists of a single or two points of a circle, is not an elementary internal system.

Formally it can be expressed so:

(ιA) Elementary internal system $=_{def}$

$$(\iota A)\{\{([\iota\iota a(*\iota A)])t\}\bullet\{[(A)\iota\overset{\cup}{A}]\succ \iota\iota a\}\} \qquad (4.35)$$

(ιA) Non-elementary internal system $=_{def}$

$$(\iota A)\{\{(\,[\iota\iota a(*\iota A)]\,)t\}\ \bullet\{([(A)\iota\overset{\cup}{A}\,] \succ \iota\iota a)n\}\} \qquad (4.36)$$

We finish the consideration of system parameters with the *substratum homogeneity-heterogeneity*. The first type includes systems that consist of homogeneous elements. As an example we can take all copies of a draft of a geometrical theorem proof. Any property of one of them is a property of the others.

Formally:

(ιA) Substratum homogeneous system $=_{def}$

$$(\iota A)\{\{(\,[a(*\iota A)]\,)t\}\ \bullet\{([(A)\iota\overset{\cup}{A}\,]*)[([(a)\iota\overset{\cup}{A}\,]*)A]\}\,\} \qquad (4.37)$$

Substratum heterogeneous system supposes that there are elements in it, which have various properties. E.g., a watch, a human body. Formally it can be expressed as:

(ιA) Substratum heterogeneous system $=_{def}$

$$(\iota A)\{\{(\,[a(*\iota A)]\,)t\}\ \bullet\{[(\iota\overset{\cup}{A})\iota\iota A]\ \bullet[(\iota\overset{\cup}{A})\iota\iota A']\}\} \qquad (4.38)$$

Note that one and the same object may be a heterogeneous and homogeneous system in accordance with their concepts and structures. So a sand heap can be considered as building material and is a homogeneous system. A different situation arises when sand is subjected to physical or chemical analysis. Such an analysis may discover the essential diversity of properties of separate grains of sand.

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